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Performance of Random Linear Network Codes Concatenated with Reed-Solomon Codes Using Turbo Decoding

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Abstract—In this paper, we consider a broadcast application for mobile terminals where the access points are connected to the source via a fixed network. Random Linear Network (RLN) coding is used in the fixed network to maximize its transmission rate and a wireless connection is used between the access points and the terminal. In order to reduce variations in the transmission delay and offer more flexibility to the terminal, we perform the network decoding in the terminal. We investigate the error correction capability of RLN coding using the ordered statistics decoding (OSD) algorithm. Taking into account the forward error correcting (FEC) code used on the wireless link, a product code is built by combining the FEC code and the RLN code and turbo decoded in the terminal. The proposed scheme efficiency is analyzed through its simulated and theoretical performance under additive white Gaussian noise (AWGN) and non frequency-selective block Rayleigh fading channels.

Index Terms—Broadcast, network coding, random linear code, ordered statistics, block turbo code, error correction

I. INTRODUCTION

In broadcast applications, the network coding [1] is studied for its advantages over traditional store-and-forward strategy. Linear network coding [1][2] resides in that the intermediate node can linearly combine its received packets (on a finite field) into one packet of the same length and send it to the following node. Each packet generated by the intermediate nodes will record in its header the information about the combination. By retrieving header information at the destination, we can establish the reverse operation (network decoding) to regenerate the original packets. This new strategy benefits from the correlation of different data flux heading to different sinks during the transmission. Different data flux can be fused and refined in an optimized way so that it will ease the passage of data through network bottlenecks. So the throughput of the network is improved and both energy and bandwidth are economized. According to [2], it is sufficient to apply linear combinations over a Galois field (GF) to achieve the capacity of the error-free broadcast networks. Later, the random linear network (RLN) coding was proposed in [3] where the linear combination coefficients are chosen randomly, uniformly and independently over a GF with large enough cardinal. This

randomness approach guarantees us a capacity-achieving configuration of network coding with a high probability that its reverse operation exists. More detailed discussion on network coding theory can be referred to [4].

In broadcast networks, the packets sent to a particular destination will travel through different routes and different delays will be introduced. Suppose that RLN coding is applied, then every packet received by the destination is a random linear combination of original packets. The destination needs to wait for at least K linearly independent packets before the network decoding. In this paper, we investigate a compromise between transmission delay and error rate performance. Fig. 1 illustrates the network where the source node S sends K packets to Access Points (AP) by broadcasting in an error-free fixed network. The RLN coding is applied between S and APs to exploit the network redundancy. The end user T is attached to AP via a wireless channel, in which, the FEC code is used for improving transmission reliability.

The GF cardinal is supposed to be sufficiently large, so AP can accomplish network decoding with high probability as long as it has received at least K independent packets. A straightforward solution is to let AP do the network decoding. After getting the original packets, the AP encodes them with a FEC code and sends them to T . The system error rate performance is then determined by the FEC code. Its drawback is that the packet delay in the broadcast network can differ a lot [5] and each AP has to wait for K independent packets before network decoding and wireless transmission. Another problem happens when T moves from AP1 to AP2 during the wireless transmission. It will only receive M packets ($M < K$) from AP1. In order to recover all K original packets, T has to resort to AP2 either by indicating the $(K - M)$ missing packet numbers through signaling channel or with some central control unit over the APs to let AP2 know the missing packets. This solution suffers from additional cost.

In this paper, we consider another solution with high flexibility when the end user switches AP during the transmission. The AP starts the wireless transmission of the network coded packets (after FEC encoding) as soon as they are received at AP. If T changes its connected AP, it simply continues to receive further network coded packets from a new AP without additional signaling nor central control. It is at T where we

decode both the channel and network codes. This solution is not tightly restricted by the packet delay in broadcast network so the transmission efficiency will be improved.

The remainder of the paper is organized as follows. In Section II, we describe the system setup and the coding scheme. In Section III, we study the error correction performance of RLN code and we introduce the ordered statistics decoding (OSD) [6]. In Section IV, we describe how to use the OSD in a turbo decoding algorithm. In Section V, the system frame error rate (FER) under AWGN and non frequency-selective block Rayleigh fading channels is analyzed. In Section VI, we conclude this paper.

II. SYSTEM DESCRIPTION

The system setup is depicted in Fig. 1. An error-free network connects the source **S** and the Access Points (AP) and RLN coding is applied in this fixed network. Without loss of generality, we suppose there are at least N disjoint paths from **S** to each AP. The end user **T** is connected to an AP via a wireless channel and can switch AP during the transmission.

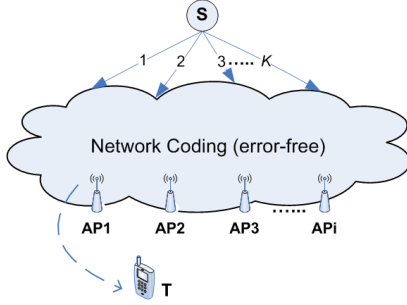


Fig. 1. One-source multicast network structure using network coding

For each transmission, **S** maps data into K packets of length K' denoted by $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_K, K < N$ on $\text{GF}(q)$ and sends them out. The network nodes perform RLN coding with coefficients chosen randomly and uniformly over the data symbol alphabet $\text{GF}(q)$. So, the output packet of an intermediate node is a random linear $\text{GF}(q)$ -combination of its received packets. Each AP receives N packets of length K' . For a particular AP, we denote $\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_N$ its received packets and we define matrices **X** and **Y** whose rows are respectively transmitted and received packets. The linear combinations from **S** to AP can be modeled by an $N \times K$ transfer matrix **F** as described in [2]. The transfer equation is:

$$\mathbf{Y} = \mathbf{F} \cdot \mathbf{X}. \quad (1)$$

According to [3], if the cardinal basis of $\text{GF}(q)$ tends to infinity, the probability that the RLN coding yields a solution tends to 1. Then the network decoding is:

$$\hat{\mathbf{X}} = (\mathbf{F}^T \mathbf{F})^{-1} \mathbf{F}^T \mathbf{Y}, \quad (2)$$

where \mathbf{F}^T denotes the transpose of **F** and \mathbf{F}^{-1} its inverse.

After detecting the original data, the AP transmits them to **T** using traditional wireless protocols (e.g. Wi-Fi). This

straightforward solution mentioned in Section I has the drawback that the transmission delay is determined by the longest route in the broadcast network. Since transmission delay is a major system concern, we consider an alternative solution.

Suppose **T** is initially connected to AP1. After AP1 has received several network coded packets, instead of waiting for the others, it encodes them using a FEC code. We consider a linear FEC code defined on $\text{GF}(q)$ (binary dimension $K'm$, binary length $N'm$) where $m = \log_2(q)$. The resulting codewords are sent to **T** immediately. When **T** moves to AP2, it just continues to receive FEC coded packets from AP2. We assume a BPSK transmission and \mathbf{z}_i is the BPSK modulated FEC codeword. Supposing an AWGN channel, let \mathbf{b}_i denote a $N'm$ -length vector with independent identically distributed (i.i.d) zero mean Gaussian components, and let \mathbf{r}_i be the i -th packet of length $N'm$ received by the end user **T**:

$$\mathbf{r}_i = \mathbf{z}_i + \mathbf{b}_i. \quad (3)$$

By stacking successively all the received packets \mathbf{r}_i ($i = 1 \dots N$), we build the received matrix **R** of dimension $N \times N'm$. In the same way, we build the transmitted $\text{GF}(q)$ matrix **Z** of dimension $N \times N'$ whose rows are the FEC codewords \mathbf{z}_i ($i = 1 \dots N$) converted to $\text{GF}(q)$.

The row vectors of matrix **Z** are the FEC codewords. We denote the i -th column vector of **X** by \mathbf{x}_i^c and its i -th row vector by \mathbf{x}_i^r . We consider a systematic FEC code whose redundancy comes first so that the last K' columns of **Z** are the result of network coding in (1) that remains unchanged during FEC encoding. For simplicity, we continue to use **Y** to represent these columns of **Z**. From (1):

$$(\mathbf{y}_i^c)^T = (\mathbf{x}_i^c)^T \cdot \mathbf{F}^T. \quad (4)$$

So the column code of **Y** can be modeled as a FEC code whose generator matrix is \mathbf{F}^T . We name this code as the RLN code. Providing the RLN code can be made systematic, a two-dimensional product code can be built by serial concatenation of the systematic RLN code and the systematic FEC code.

To transform \mathbf{F}^T into its systematic form, we only need to apply the row elementary operation and column permutation on \mathbf{F}^T . It leads to a permutation of the column codeword elements (a row permutation on **Z**). So at the end user side, after arranging the rows of **R** according to the permutation applied to **Z**, we will get a product codeword, whose row code is the systematic FEC code used at APs and its column code is a random linear block code whose generator matrix is the systematic form of \mathbf{F}^T . This permutation is invertible after decoding. In the following, for the sake of simplicity, we keep the notations **R**, **Z** and \mathbf{F}^T to denote the resulting matrices after the systematic transformation.

III. ORDERED STATISTICS DECODING (OSD) AND RANDOM LINEAR NETWORK CODE

Some researchers have studied the error correction capability of network coding in erroneous networks. Refs. [7]-[10] introduce different theories about the error-control in Network

Coding. These emerging theories analyze the problem within the vector space domain and they handle the network code through hard decoding approach and the reliability information is not exploited.

In this paper, we investigate the soft decoding of RLN code. For soft decoding, we use the *log-likelihood ratio* (LLR) as a measure of the reliability. Under AWGN channel and BPSK modulation assumption, the decoder input is:

$$LLR_{ij}^{in} = \ln \frac{\Pr\{z_{ij} = +1/r_{ij}\}}{\Pr\{z_{ij} = -1/r_{ij}\}}. \quad (5)$$

As described in Section II, the network coding transfer matrix \mathbf{F} is updated for each transmission of a data matrix \mathbf{X} . The corresponding RLN code doesn't have a fixed minimum Hamming distance. Decoding this kind of code with ordinary decoding methods is not efficient due to its variable generator matrix. So we propose to use the soft ordered statistics decoding (OSD) algorithm. It was proposed in [6] to decode a binary linear block code depending on the reliability measure. Here is a brief review.

The linear codeword is symbol-to-binary converted (systematic or not) with binary code length and code dimension equal to N_b and K_b , respectively. We reorder the bits of received codeword \mathbf{c}_1 according to their reliabilities in decreasing order and get \mathbf{c}_2 . The reordering defines a permutation function λ_1 to be applied on the columns of generator matrix \mathbf{G} , resulting in matrix \mathbf{G}' . We find the first K_b independent columns with highest associated reliabilities in \mathbf{G}' and put them as the first K_b columns of a matrix \mathbf{G}'' of dimension $K_b \times N_b$. The remaining $(N_b - K_b)$ columns of \mathbf{G}' complete \mathbf{G}'' . This selection defines a permutation function λ_2 to be used on \mathbf{c}_2 getting a sequence \mathbf{c}_3 . The first K_b components of \mathbf{c}_3 are called its most reliable independent (MRI) bits. With the MRI bits as message part and the corresponding systematic generator matrix \mathbf{G}'' , we get a hard decision codeword $\hat{\mathbf{a}}$. Its corresponding original codeword can be obtained by applying permutation $\lambda_1^{-1} \lambda_2^{-1}$. The *order-1 reprocessing* aims at making all possible changes of i bits ($1 \leq i \leq l$) among the K_b MRI bits and re-encoding them into candidate codewords with the generator matrix \mathbf{G}'' . Finally, we choose the most probable one as the decoding result.

The OSD is compatible with the variable generator matrix of RLN code without much additional complexity. Here is an example of *order-1* OSD to decode a RLN code of dimension 5 and code length form 7, 8 to 9 on GF(8). In Table I, we build a statistics by generating a large number of such random codes and classify them according to their minimum Hamming distances (d_{\min}).

TABLE I
MINIMUM DISTANCE DISTRIBUTION OF RANDOM CODE (N,5) ON GF(8)

Min distance (bit)	1	2	3	4	5
Code(7,5)	10.5%	77.1%	12.4%	0	0
Code(8,5)	1.53%	24%	68.07%	6.4%	0
Code(9,5)	0.2%	4.5%	32.2%	60.2%	2.9%

We shall now calculate the union bound of the FER as:

$$\text{FER}|_{d_{\min}=k} \leq \frac{1}{2} \sum_{d=k}^n \bar{W}_d|_{d_{\min}=k} \cdot \text{erfc} \left(\sqrt{d R_a \frac{E_b}{N_0}} \right). \quad (6)$$

$$\text{FER} = \sum_{d_{\min}=1}^{\max(d_{\min})} (\text{FER}|_{d_{\min}=k}) \cdot \Pr(d_{\min} = k), \quad (7)$$

where $\text{FER}|_{d_{\min}=k}$ is the average FER in condition that all the codes tested satisfy $d_{\min} = k$ and \bar{W}_d is the average multiplicity of codewords with weight d . $\text{erfc}(x)$ is the complementary error function and R_a is the code rate. The bounds and simulated FER of the RLN codes are illustrated in Fig. 2.

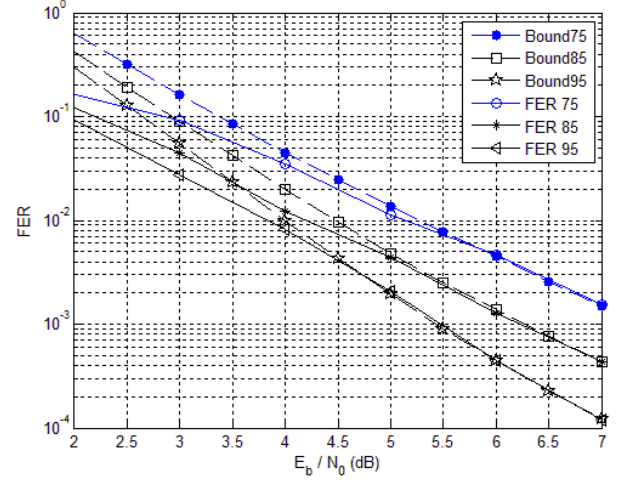


Fig. 2. Union bounds / FER simulated of OSD(1) on GF(8) in AWGN channel with BPSK modulation

We observe that the *order-1* OSD successfully decodes such random codes as the simulated FER reaches its union bounds (in the three cases, both curves are superposed for FER less than 4×10^{-3}). Moreover, simulated results match with Table I: with each unity increase of code length, the percent of $d_{\min} = 1$ has decreased by a factor of about 10, resulting in an improvement of the error rates. $d_{\min} = 1$ is to be avoided as it means no error correction capability.

IV. ITERATIVE DECODING AT DESTINATION

Since we can piece together the received packets into a product code, we will apply turbo decoding at the end user. The turbo decoder receives a noisy BPSK modulated $N \times N'm$ matrix \mathbf{R} corresponding to the product codeword whose row code is a linear block FEC code and its column code is a RLN code. The Soft-in-Soft-out (SISO) decoding is performed iteratively on \mathbf{R} using the Chase-Pyndiah algorithm [12] reinforced by the OSD for the RLN code. For the i -th row of \mathbf{R} : $\mathbf{r}_i^T = (r_{i1}, r_{i2}, \dots, r_{in})$ where $n = N'm$, the Chase algorithm [11] generates a subset Ω_1 of the most probable row codewords. Within Ω_1 , the most probable codeword is noted $\mathbf{d} = (d_1, d_2, \dots, d_n), d_k \in \{\pm 1\}$. We define the competing codeword $\mathbf{c}^{(j)} = (c_1, c_2, \dots, c_n), c_k \in \{\pm 1\}$, for a specific

position j as the most probable codeword in Ω_1 with $c_j^{(j)} \neq d_j$. Then the soft output LLR of the decision d_j is calculated as:

$$LLR_{ij}^{out} = \frac{\langle \mathbf{LLR}_i^{in}, \mathbf{d} \rangle - \langle \mathbf{LLR}_i^{in}, \mathbf{c}^{(j)} \rangle}{2} \cdot d_j, \quad (8)$$

where $\langle \mathbf{a}, \mathbf{b} \rangle = \sum_k a_k b_k$ is the correlation between two vectors \mathbf{a} and \mathbf{b} . The extrinsic information is obtained by:

$$W_{ij} = LLR_{ij}^{out} - LLR_{ij}^{in}. \quad (9)$$

If there is no competing codeword $\mathbf{c}^{(j)}$ in Ω_1 , then we use:

$$W_{ij} = \beta \times d_j, \quad (10)$$

where β is a constant to be optimized through simulations.

For the columns of \mathbf{R} , we use the SISO OSD [13]. The *order-1* reprocessing is used to generate the candidate column codeword subset Ω_2 to find the competing codeword. The rest of the column decoding procedure is the same as the row one.

We update the input LLR every half iteration by:

$$\mathbf{LLR}^{in}(m+1) = \mathbf{LLR}^{in}(0) + \alpha(m+1) \cdot \mathbf{W}(m), \quad (11)$$

where the $\mathbf{LLR}^{in}(0)$ is the channel output LLR and $\mathbf{W}(m)$ is the extrinsic information obtained at the previous iteration. At the first iteration, $\mathbf{W}(0)$ is set to zero. $\alpha(m)$ is a scaling factor increasing with the iteration step m .

V. SIMULATION RESULTS

For each transmission of a data matrix \mathbf{X} , the matrix \mathbf{F} is constructed randomly. \mathbf{T} will test if \mathbf{F} has full rank. An additional packet will be requested until having K independent ones. The more packets are received by \mathbf{T} , the more probable \mathbf{F} is invertible. Another interest of increasing the packets received by \mathbf{T} is to improve the d_{\min} distribution of the RLN code as shown in Table I and Fig. 2.

We investigate the performance of our solution on AWGN and non frequency-selective block Rayleigh fading channels using BPSK modulation. At destination, we suppose perfect synchronization, no intersymbol interference (ISI), perfect channel information and knowledge of the transfer matrix \mathbf{F} . Our simulation is based on GF(32). The FEC code is the Reed-Solomon (RS) code(31, 29). Different sizes of generator matrix \mathbf{F}^T with $K = 5, N = 7, 8$ or 9 are tested.

The Chase algorithm considers the 4 least reliable bits whose indices compose the subset Θ . For an 8-iteration turbo decoding, we use the following scaling factor α and reliability factor β for the different half-iterations:

$$\alpha = \{0, 0.1, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.9, 1.0, 1.0\}.$$

$$\beta = \min \left(\sum_{j \in \Theta} |r_j^{in}|, 5 \right).$$

$|r_j^{in}|$ is the reliability of the j -th least reliable input binary component. For the *order-1* reprocessed OSD, we choose the

competing codeword among the 6 most probable codewords in Ω_2 to eliminate possible pseudo-competing codewords.

Here, we consider the FER without the error propagation due to network decoding that will influence the BER. Fig. 3 plots the FER in AWGN channel. The curve “RS(31, 29) soft” is the reference representing the soft decoding of RS(31, 29) using Chase algorithm with 16 test patterns. It can be viewed as the straightforward solution where network decoding is applied at **AP** before transmitting all the RS encoded original packets to **T**. As pointed out in Section II, this solution will suffer from long delays. For the proposed (7, 5) network code based scheme, the SNR gain is equal to 2dB for a FER of 10^{-2} and increases with the number of packets received by **T**: 2.5dB (resp. 2.75dB) for $N = 8$ (resp. $N = 9$).

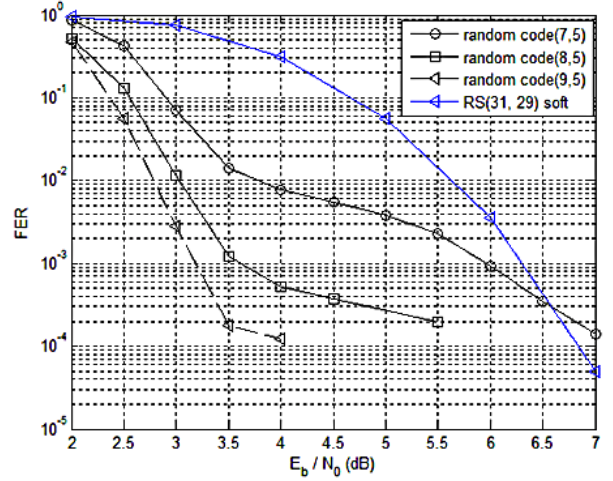


Fig. 3. Proposed scheme performance on AWGN channel, BPSK / GF(32)

In Fig. 3, we observe an error floor phenomenon for our proposed solution especially for high SNR. We analyze it using Fig. 4 where we consider only the case of RLN code (7, 5). The curve “Turbo (7, 5) $d_{\min} = 1$ ” (resp. “Turbo (7, 5) $d_{\min} > 1$ ”) is the FER of our proposed solution when the RLN code has $d_{\min} = 1$ (resp. $d_{\min} > 1$). The overall average FER curve for RLN code (7, 5) with all possible minimum distances is the one with diamond marks.

Based on statistics tests, we estimate $p = \Pr(d_{\min} = 1) \approx 6.8 \times 10^{-3}$ for the RLN code (7, 5) on GF(32), and use the approximation of the FER given in (12):

$$\begin{aligned} \text{FER} &= \sum_{d_{\min}=1}^{\max(d_{\min})} (\text{FER}|_{d_{\min}=k}) \cdot \Pr(d_{\min} = k) \\ &= \text{FER}|_{d_{\min}=1} \cdot p + \text{FER}|_{d_{\min}>1} \cdot (1 - p). \end{aligned} \quad (12)$$

The condition d_{\min} refers to the minimum Hamming distance of the random component code in the product code. Since $\text{erfc}(\sqrt{x})$ decreases exponentially with x , from (6) and (12), at low SNR where $\text{FER}|_{d_{\min}=1} \approx 1$ and $\text{FER}|_{d_{\min}>1} \gg p$:

$$\text{FER} \approx 1 \cdot p + \text{FER}|_{d_{\min}>1} \approx \text{FER}|_{d_{\min}>1}. \quad (13)$$

It corresponds to the superposition of the overall FER curve and the curve “Turbo (7, 5) $d_{\min} > 1$ ” below 3.5dB in Fig. 4.

At high SNR, $\text{FER}|_{d_{\min}=1}$ cannot be approximated by 1 and $\text{FER}|_{d_{\min}>1}$ becomes negligible compared to $p \cdot \text{FER}|_{d_{\min}=1}$:

$$\text{FER} \approx p \cdot \text{FER}|_{d_{\min}=1} + \text{FER}|_{d_{\min}>1} \approx p \cdot \text{FER}|_{d_{\min}=1}. \quad (14)$$

The factor p in (14) corresponds to the vertical gap at high SNR in Fig. 4 between the overall average FER curve and the “Turbo (7,5) $d_{\min} = 1$ ” curve. This factor causes the error floor effect. If it was possible to eliminate the configuration $d_{\min} = 1$, the error floor effect would disappear as shown by the curve with pentagrams. But since the network coding is randomly generated by the network nodes, there is no evident solution. An intermediate solution is to check if the network configuration leads to a RLN code with $d_{\min} = 1$ and request additional packets if necessary. We observe that the percent of RLN code (7,5) having $d_{\min} = 1$ decreases with the Galois field cardinal q (10% for $q = 8$ against 0.7% for $q = 32$ and 0.17% for $q = 64$). It may be interesting to deeply investigate the influence of q over d_{\min} .

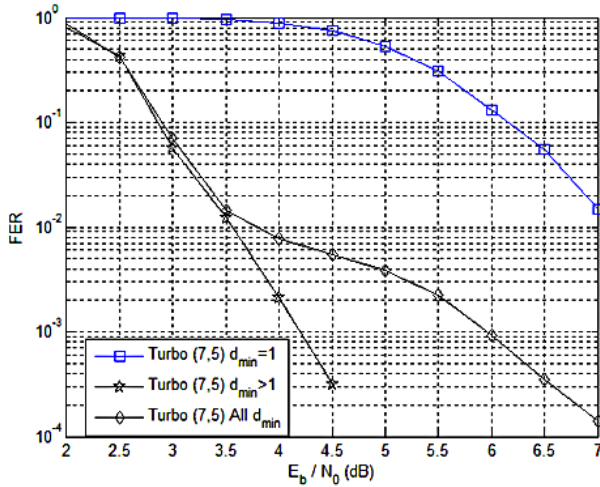


Fig. 4. Proposed scheme performance on AWGN channel, with a (7,5) network code, BPSK and GF(32)

Fig. 5 plots the FER for a flat Rayleigh block fading channel. Each row of the product code is attenuated with an independent Rayleigh coefficient. At 10^{-2} , the coding gain compared to the AP network decoding solution is about 8dB with a RLN code (7,5). There is an average improvement of 2.5dB per additional received packet.

VI. CONCLUSION

In this paper, we propose a flexible transmission scheme for the broadcast network where network coding is applied. As the random linear network code can be modeled as a random linear block FEC code, it can be softly decoded by the ordered statistics decoding algorithm. Simulations proved that, the more coded packets collected from the network, the better the error correction performance. Increasing the received packet number ameliorates the minimum Hamming distance distribution of the resulting random code. The error floor

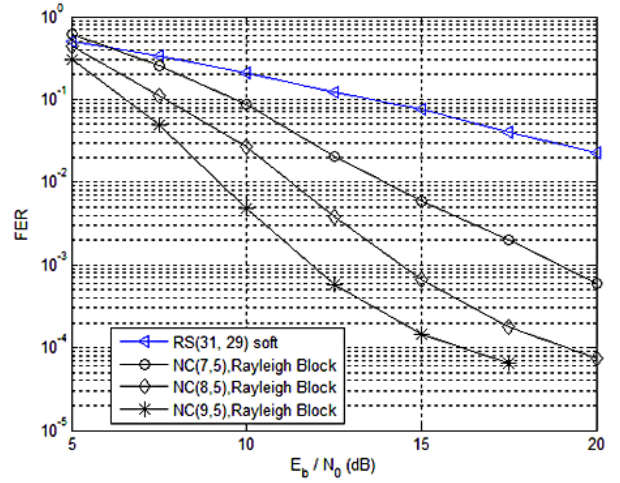


Fig. 5. Proposed scheme performance on Rayleigh block fading channel with BPSK and GF(32)

effect is analyzed using the union bounds and one possible solution is to avoid the network coding configurations leading to a minimum Hamming distance of 1.

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